Solution by the Proposer.

As this triangle is designed to show the slope of earth embankments the word 'slope' is defined as it is understood by the civil engineer: the ratio of horizontal to vertical distance.

Let AD, BE, CF be the medians of the required triangle, ABC, then

$$(1)\qquad 2\cot ADB = \cot C - \cot B,$$

$$(2)\qquad 2\cot BEC = \cot A - \cot C,$$

$$(3)\qquad 2\cot CFA = \cot B - \cot A,$$

and hence $\cot ADB + \cot BEC + \cot CFA = 0$. If we take $\cot ADB = \frac{2}{3}$ then from the assigned values: (a) $\cot BEC = -1$, $\cot CFA = \frac{1}{3}$; (b) $\cot BEC = \frac{1}{3}$, $\cot CFA = -1$. Considering case (a), these values inserted in (1), (2), (3) give two independent equations, which are to be combined with the identity,

$$(4)\qquad \cot B \cot C + \cot C \cot A + \cot A \cot B = 1.$$

Putting, from (2) and (3), $\cot B = \cot A + \frac{2}{3}$, $\cot C = \cot A + 2$, equation (4) becomes

$$9\cot^2 A + 16\cot A + 1 = 0;$$

whence,

$$\cot A = \frac{-8 \pm \sqrt{55}}{9}; \qquad \cot B = \frac{-2 \pm \sqrt{55}}{9}; \qquad \text{and} \qquad \cot C = \frac{10 \pm \sqrt{55}}{9}.$$

Since a triangle can have only one obtuse angle only one cotangent can be negative. Consequently, the ($-$) before the radical does not lead to a solution. Using the ($+$) sign,

$$A = 93° 42' 42'', \qquad B = 58° 57' 37'', \qquad C = 27° 19' 41''.$$

When $c = 10.00$, $a = 21.74$ and $b = 18.66$.

Case (b) can be treated in the same way and we get

$$\cot A = \frac{8 + \sqrt{55}}{9}, \qquad \cot B = \frac{-10 + \sqrt{55}}{9}, \qquad \cot C = \frac{2 + \sqrt{55}}{9}.$$

2745 [1919, 37]. Proposed by G. I. HOPKINS, Manchester, N. H.

A recent English publication contains the following method of constructing a regular polygon of 17 sides: Draw the radius CB perpendicular to the diameter AQ of the circle whose center is B. On BC lay off BD equal to one-fourth of BC. On BA, lay off BE and draw DE making angle BDE one fourth of angle BDA. On BQ lay off BF and draw DF, making angle FDE 45°. On AF as diameter, construct semi-circle FHA intersecting CB in H. With E as center and EH as radius construct semi-circle LHK intersecting CB in H. At the points L and K draw the ordinates NL and MK. Bisect the arc NM and let P be the point of bisection. Then the chord $NP(= MP)$ is a side of the regular polygon of 17 sides. Is the method of construction correct?

I. Solution by C. H. Chepmell, Hove, England.

The abscissas BK, BL, and their ordinates to M and N, make the angles MBA, NBA equal to $10\pi/17$, and $6\pi/17$ respectively. Consequently the difference of these two angles, the angle MBN, is equal to $4\pi/17$. This justifies the claim of the construction.

But tables of all the trigonometrical functions of $n\pi/17$ are not readily available; and it may be more satisfactory if we outline the connection between the numerical value of NM and the numerical value of some function given in a standard work.

Taking the radius of the circle ACQ as unity, we find

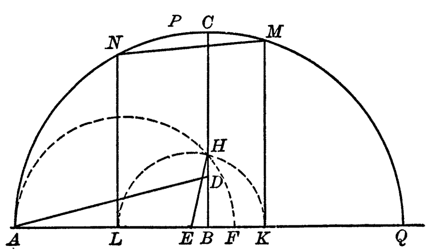
$$BE = \frac{-1 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}}}{16},$$

$$BF = \frac{+1 - \sqrt{17} + \sqrt{34 - 2\sqrt{17}}}{16},$$

$$EH = \frac{\sqrt{2(\sqrt{17} - 3)(2\sqrt{17} + \sqrt{34 + 2\sqrt{17}})}}{16} = \frac{\beta}{16},$$

$$AK = \frac{+17 + \sqrt{17} - \sqrt{34 + 2\sqrt{17}} + \beta}{16},$$

$$AL = \frac{+17 + \sqrt{17} - \sqrt{34 + 2\sqrt{17}} - \beta}{16},$$



$$KQ = \frac{+15 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}} - \beta}{16},$$

$$LQ = \frac{+15 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}} + \beta}{16},$$

$$LN = \sqrt{AL \cdot LQ} = \tfrac{1}{8}\sqrt{+34 + 2\sqrt{17} + 2\sqrt{34 + 2\sqrt{17}} - 2\beta'},$$

$$KM = \sqrt{AK \cdot KQ} = \tfrac{1}{8}\sqrt{+34 + 2\sqrt{17} + 2\sqrt{34 + 2\sqrt{17}} + 2\beta'},$$

$$[\beta' = \sqrt{2(\sqrt{17} - 3)(2\sqrt{17} - \sqrt{34 + 2\sqrt{17}})}; \quad \text{and} \quad \beta \times (-1 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}}) = 4 \cdot \beta'],$$

$$LN \times KM = \tfrac{4}{64}\sqrt{2(\sqrt{17} + 3)(2\sqrt{17} + \sqrt{34 - 2\sqrt{17}})} = \frac{4\alpha}{64},$$

$$\begin{aligned} NM^2 &= LK^2 + (KM - LN)^2 \\ &= 4 \cdot EH^2 + KM^2 + LN^2 - \frac{8\alpha}{64} \\ &= \frac{+136 - 8\sqrt{17} + 8\sqrt{34 - 2\sqrt{17}} - 8\alpha}{64}, \end{aligned}$$

$$[\alpha' = \sqrt{2(\sqrt{17} + 3)(2\sqrt{17} - \sqrt{34 - 2\sqrt{17}})}; \quad \text{and} \quad 4\alpha = \alpha' \times (-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}})],$$

$$\therefore \quad NM^2 = \frac{+136 - 8\sqrt{17} + 8\sqrt{34 - 2\sqrt{17}} - 2(-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}})\alpha'}{64}.$$

And, in the circle with radius equal to unity, NM represents the value $2\sin(2\pi/17) \times 1$; and therefore

$$\begin{aligned} 4\cos^2\frac{2\pi}{17} &= 4 - NM^2 \\ &= \frac{+120 + 8\sqrt{17} - 8\sqrt{34 - 2\sqrt{17}} + 2(-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}})\alpha'}{64} \\ &= \left[\frac{-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}} + \alpha'}{8}\right]^2 \end{aligned}$$

and

$$2\cos\frac{2\pi}{17} = \frac{-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}} + \sqrt{2(\sqrt{17} + 3)(2\sqrt{17} - \sqrt{34 - 2\sqrt{17}})}}{8}.$$

And this value of $2\cos(2\pi/17)$ will be found to agree with that given in Klein's *Famous Problems in Elementary Geometry* (Beman & Smith), though our α' is there written in a different form.

II. Historical Note by R. C. Archibald, Brown University.

This method of construction is due to H. W. Richmond, *Quarterly Journal of Mathematics*, Volume 26, 1893, pp. 206–207; and *Mathematische Annalen*, Volume 67, 1909, pp. 460–461. It is reproduced on page 34 of H. P. Hudson's *Ruler and Compasses*, London, 1916.

Various constructions of the regular polygon of seventeen sides were reviewed by R. Goldenring in his *Die elementargeometrischen Konstruktionen des regelmässigen Siebzehnecks* (Leipzig, 1915), but many omissions in this professedly complete survey were noted by the writer in the *Bulletin of the American Mathematical Society*, vol. 22, 239-246. The first solution in an English publication, given by Lowry in 1819,[1] was reproduced in this *Monthly*[2] in 1899 and 1914. Other solutions and historical notes are set forth in the articles printed above, pages 322–326.

2767 [1919, 171]. Proposed by W. W. JOHNSON, U. S. Naval Academy.

Let the complex quantities p, q, and r satisfy the relation $p^2 + q^2 + r^2 = 0$; prove that the corresponding vectors OP, OQ, and OR are such that if any two of them are taken as conjugate semi-diameters of an ellipse, the third lies on the minor axis, and its length is the distance from the center to either focus.

Solution by A. Pelletier, Montreal, Can.

Let $(x^2/a^2) + (y^2/b^2) = 1$, be the equation of the ellipse having OP and OQ for conjugate semi-diameters ($2a$ and $2b$ being the axes, and $a \geqq b$). If α, α', α'' are the respective arguments of

[1] *The Mathematical Repository*, new series, vol. 4, p. 160; Lowry's proof occupies pages 160-168.
[2] Volume 6, p. 239 and volume 21, p. 252.